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POISSON-LIE T-DUALITY AND N=2 SUPERCONFORMAL WZNW MODELS ON COMPACT GROUPS.

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Abstract

The supersymmetric generalization of Poisson-Lie T-duality in N=2 superconformal WZNW models on the compact groups is considered. It is shown that the role of Drinfeld's doubles play the complexifications of the corresponding compact groups. These complex doubles are used to define the natural actions of the isotropic subgroups forming the doubles on the group manifolds of the N=2 superconformal WZNW models. The Poisson-Lie T-duality in N=2 superconformal $U(2)$-WZNW model considered in details. It is shown that this model admits Poisson-Lie symmetries with respect to the isotropic subgroups forming Drinfeld's double $Gl(2,C)$. Poisson-Lie T-duality transformation maps this model into itself but acts nontrivialy on the space of classical solutions. Supersymmetric generalization of Poisson-Lie T-duality in N=2 superconformal WZNW models on the compact groups of higher dimensions is proposed.

Introduction.

Target space duality in string theory has attracted a considerable attention in recent years because it sheds some light on the geometry and symmetries of string theory. The well known example of T-duality is mirror symmetry in the Calaby-Yao manifolds compactifications of the superstring [1]. Duality symmetry was first described in the context of toroidal compactifications [2]. For the simplest case of single compactified dimension of radius R, the entire physics of interacting theory is left unchanged under the replacement $R \to \alpha/R$ provided one also transforms the dilaton field $\phi \to \phi - \ln(R/\sqrt{\alpha})$ [3]. The T-duality symmetry was later extended to the case of nonflat conformal backgrounds with some abelian isometry (abelian T-duality) in [4].

The basic idea of the notion of non-abelian duality [5, 6] is to consider a conformal

field theory with a non-abelian symmetry group. The non-abelian duality did miss a lot of features characteristic to the abelian duality. For example the non-abelian T-duality transformation of the isometric σ- model on a group manifold G gives non-isometric σ-model on its Lie algebra [5, 7]. As a result, it was not known how to perform the inverse duality transformation to get back to the original model.

A solution of this problem was proposed recently in [8], where it was argued that the two theories are dual to each other from the point of view of the so called Poisson-Lie (PL) T-duality. The main idea of their approach is to replace the requirement of isometry by a weaker condition which is the Poisson-Lie symmetry of the theory. This generalized duality is associated with two groups forming a Drinfeld double [9] and the duality transformation exchanges their roles. This approach has recieved futher developments in the series of works [10], [11], [12], [13], [14].

In order to apply PL T-duality in superstring theory one needs to have the dual pairs of conformal and superconformal σ-models. The simple example of dual pair of conformal σ-models associated with the $O(2,2)$ Drinfeld double was presented in work [15]. Then, it was shown in [11], [12] that WZNW models on the compact groups are the natural examples of PL dualizable σ-models.

The supersymmetric generalization of PL T-duality was considered in [16, 17]. In particular, due to the close relation between N=2 superconformal WZNW models and Drinfeld's double (Manin triple) structures on the corresponding group manifolds (Lie algebras) [18, 19], it was shown in the paper [17] that N=2 superconformal WZNW models admit very natural PL symmetry and PL T-dual σ-models for N=2 WZNW models associated with real Drinfeld's doubles was constructed.

In the present note we consider PL T-duality in N=2 superconformal WZNW models on the compact groups. These models correspond to the complex Manin triples endowed with hermitian conjugation which conjugates isotropic subalgebras of the Manin triple.

After a brief review of the classical N=1 superconformal WZNW models in the section 1, we consider in the section 2, the Manin triple construction of N=2 superconformal WZNW models on the compact groups and define the action of isotropic subgroups from the correspondig Drinfeld's doubles on the space of fields. In the section 3 we consider N=2 superconformal $U(2)$-WZNW model. We shall show by the direct calculations that this model admits Poisson-Lie symmetries with respect to the action of the isotropic subgroups forming Drinfeld's double. Using these symmetries we show that Poisson-Lie T-dual σ-model is N=2 $U(2)$-WZNW model itself. We conclude with some proposals about extensions of our results for N=2 superconformal WZNW models on the other compact groups and Kazama-Suzuki models.

1. The classical N=1 superconformal WZNW model.

In this section we briefly review a superconformal WZNW (SWZNW) models using superfield formalism [20].

We parametrize super world-sheet introducing the light cone coordinates $x_{\pm}$, and grassman coordinates $\Theta_{\pm}$. The generators of supersymmetry and covariant derivatives are

$$Q_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} + \imath \Theta_{\pm} \partial_{\mp}, \; D_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} - \imath \Theta_{\pm} \partial_{\mp}. \tag{1}$$

They satisfy the relations

$$\{D_\pm, D_\pm\} = -\{Q_\pm, Q_\pm\} = -\imath 2\partial_\pm,\ \{D_\pm, D_\mp\} = \{Q_\pm, Q_\mp\} = \{Q, D\} = 0, \tag{2}$$

where the brackets $\{,\}$ denote the anticommutator. The superfield of N=1 supersymmetric WZNW model

$$G = g + \imath\Theta_-\psi_+ + \imath\Theta_+\psi_- + \imath\Theta_-\Theta_+ F \tag{3}$$

takes values in a Lie group **G**. We will assume that its Lie algebra **g** is endowed with ad-invariant nondegenerate inner product $<,>$.

The inverse group element G^{-1} is defined from the relation

$$G^{-1}G = 1 \tag{4}$$

and has the decomposition

$$G^{-1} = g^{-1} - \imath\Theta_- g^{-1}\psi_+ g^{-1} - \imath\Theta_+ g^{-1}\psi_- g^{-1} - \imath\Theta_-\Theta_+ g^{-1}(F + \psi_- g^{-1}\psi_+ - \psi_+ g^{-1}\psi_-)g^{-1} \tag{5}$$

The action of N=1 SWZNW model is given by

$$S_{swz} = \int d^2x d^2\Theta(< R_+, R_- >)$$
$$-\int d^2x d^2\Theta dt < G^{-1}\frac{\partial G}{\partial t}, \{R_-, R_+\} >, \tag{6}$$

where

$$R_\pm = G^{-1}D_\pm G. \tag{7}$$

The classical equations of motion can be obtained by making a variation of (6):

$$\delta S_{swz} = \int d^2x d^2\Theta < G^{-1}\delta G, D_- R_+ - D_+ R_- - \{R_-, R_+\} > \tag{8}$$

Taking into account kinematic relation

$$D_+ R_- + D_- R_+ = -\{R_+, R_-\} \tag{9}$$

we obtain

$$D_- R_+ = 0 \tag{10}$$

The action (6) is invariant under the super-Kac-Moody and N=1 superconformal transformations [20].

In the following we will use supersymmetric version of Polyakov-Wiegman formula [21]

$$S_{swz}[GH] = S_{swz}[G] + S_{swz}[H] + \int d^2x d^2\Theta < G^{-1}D_+ G, D_- H H^{-1} > . \tag{11}$$

It can be proved as in the non supersymmetric case.

2. N=2 superconformal WZNW models on compact groups.

In works [22, 18, 19] supersymmetric WZNW models which admit extended supersymmetry were studied and correspondence between extended supersymmetric WZNW

models and finite-dimensional Manin triples was established in [18, 19]. By the definition [9], a Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ consists of a Lie algebra $\mathbf{g}$, with nondegenerate invariant inner product $<,>$ and isotropic Lie subalgebras $\mathbf{g}_\pm$ such that $\mathbf{g} = \mathbf{g}_+ \oplus \mathbf{g}_-$ as a vector space.

The corresponding Sugawara construction of N=2 Virasoro superalgebra generators was given in [18, 19, 22, 23].

To make a connection between Manin triple construction of [18, 19] and approach of [22] based on complex structures on Lie algebras the following comment is relevant.

Let $\mathbf{g}$ be a real Lie algebra and J be a complex structure on the vector space $\mathbf{g}$. J is referred to as the complex structure on the Lie algebra $\mathbf{g}$ if J satisfies the equation

$$[Jx, Jy] - J[Jx, y] - J[x, Jy] = [x, y] \tag{12}$$

for any elements x, y from $\mathbf{g}$. It is clear that corresponding Lie group is complex manifold with left (or right) invariant complex structure. In the following we shall denote a real Lie group and real Lie algebra with a complex structure satisfying (12) as the pairs $(\mathbf{G}, J)$ and $(\mathbf{g}, J)$ correspondingly.

Suppose the existence of a nondegenerate invariant inner product $<,>$ on $\mathbf{g}$ so that the complex structure J is skew-symmetric with respect to $<,>$. In this case it is not difficult to establish the correspondence between complex Manin triples and complex structures on Lie algebras. Namely, for each complex Manin triple $(\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-)$ exists a canonic complex structure on the Lie algebra $\mathbf{g}$ such that subalgebras $\mathbf{g}_\pm$ are its $\pm\imath$ eigenspaces. On the other hand, for each real Lie algebra $\mathbf{g}$ with nondegenerate invariant inner product and skew-symmetric complex structure J on this algebra one can consider the complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$. Let $\mathbf{g}_\pm$ be $\pm\imath$ eigenspaces of J in the algebra $\mathbf{g}^{\mathbb{C}}$ then $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_+, \mathbf{g}_-)$ is a complex Manin triple . Moreover it can be prooved [18] that there exists one-to-one correspondence between the complex Manin triple endowed with antilinear involution which conjugates isotropic subalgebras $\tau : \mathbf{g}_\pm \to \mathbf{g}_\mp$ and the real Lie algebra endowed with ad-invariant nondegenerate inner product $<,>$ and the complex structure J which is skew-symmetric with respect to $<,>$. Therefore we can use this conjugation to extract a real form from the complex Manin triple.

If the complex structure on the Lie algebra is fixed then it defines the second supersymmetry transformation [22].

In this paper we concentrate on N=2 SWZNW models on the compact groups, that is we shall consider complex Manin triples such that the corresponding antilinaer involutions will coincide with the hermitian conjugations. Hence it will be implied in the following that $\mathbf{G}$ is a subgroup in the group of unitary matrices and the matrix elements of the superfield G satisfy the relations:

$$\bar{g}^{mn} = (g^{-1})^{nm},\ \bar{\psi}^{mn}_{\pm} = (\psi^{-1})^{nm}_{\pm},\ \bar{F}^{mn} = (F^{-1})^{nm}, \tag{13}$$

where we have used the following notations

$$\psi^{-1}_{\pm} = -g^{-1}\psi_{\pm}g^{-1},\ F^{-1} = -g^{-1}(F + \psi_- g^{-1}\psi_+ - \psi_+ g^{-1}\psi_-)g^{-1}. \tag{14}$$

Now we have to consider some geometric properties of the N=2 SWZNW models closely related with the existence of a complex structure J on the corresponding groups.

Let's fix some compact Lie group with a complex structure $(\mathbf{G}, J)$ and consider its Lie algebra with the complex structure $(\mathbf{g}, J)$. The complexification $\mathbf{g}^{\mathbb{C}}$ of $\mathbf{g}$ has the Manin triple structure $(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_{+}, \mathbf{g}_{-})$. The Lie group version of this triple [24, 25, 26] is the double Lie group $(\mathbf{G}^{\mathbb{C}}, \mathbf{G}_{+}, \mathbf{G}_{-})$ [25], where the exponential groups $\mathbf{G}_{\pm}$ correspond to the Lie algebras $\mathbf{g}_{\pm}$. The real Lie group $\mathbf{G}$ is extracted from its complexification with help of hermitian conjugation τ

$$\mathbf{G} = \{g \in \mathbf{G}^{\mathbb{C}} | \tau(g) = g^{-1}\} \tag{15}$$

Each element $g \in \mathbf{G}^{\mathbb{C}}$ from the vicinity of the unit element from $\mathbf{G}^{\mathbb{C}}$ admits two decompositions

$$g = g_{+} g_{-}^{-1} = \tilde{g}_{-} \tilde{g}_{+}^{-1}, \tag{16}$$

where $\tilde{g}_{\pm}$ are dressing transformed elements of $g_{\pm}$ [25]:

$$\tilde{g}_{\pm} = (g_{\pm}^{-1})^{g_{\mp}} \tag{17}$$

Taking into account (15) and (16) we conclude that the element g (from the vicinity of unit element from $\mathbf{G}^{\mathbb{C}}$) belongs to $\mathbf{G}$ iff

$$\tau(g_{\pm}) = \tilde{g}_{\mp}^{-1} \tag{18}$$

These equations mean that we can parametrize the elements from $\mathbf{G}$ by the elements from the complex group $\mathbf{G}_{+}$ (or $\mathbf{G}_{-}$) e.i. we can introduce complex coordinates (they are just matrix elements of g_{+} (or g_{-})) in the vicinity of unit element from $\mathbf{G}$. To do it one needs to solve with respect to g_{-} the following equation:

$$\tau(g_{-}) = (g_{+})^{g_{-}^{-1}} \tag{19}$$

(to introduce $\mathbf{G}_{-}$-coordinates on $\mathbf{G}$ one needs to solve with respect to g_{+} the equation

$$\tau(g_{+}) = (g_{-})^{g_{+}^{-1}}). \tag{20}$$

For the N=2 SWZNW model on the group $\mathbf{G}$ we obtain from (16) the decompositions for the superfield (4) (which takes values in the vicinity of unit element from $\mathbf{G}$)

$$G(x_{+}, x_{-}) = G_{+}(x_{+}, x_{-}) G_{-}^{-1}(x_{+}, x_{-}) = \tilde{G}_{-}(x_{+}, x_{-}) \tilde{G}_{+}^{-1}(x_{+}, x_{-}) \tag{21}$$

Due to (21), (11) and the definition of Manin triple we can rewrite the action (6) for this superfield in the following manifestly real form

$$S_{swz} = -\frac{1}{2} \int d^2x d^2\Theta (< \rho_{+}^{+}, \rho_{-}^{-} > + < \tilde{\rho}_{+}^{-}, \tilde{\rho}_{-}^{+} >) \tag{22}$$

where the superfields

$$\rho^{\pm} = G_{\pm}^{-1} D G_{\pm}, \; \tilde{\rho}^{\pm} = \tilde{G}_{\pm}^{-1} D \tilde{G}_{\pm} \tag{23}$$

correspond to the left invariant 1-forms on the groups $\mathbf{G}_{\pm}$ and $\tilde{\rho}^{\pm}$, ρ^{-} are expressed in terms of matrix elements of G_{+} (and its complex conjugated) with help of the formulas (16, 18, 19).

To generalize (16), (18) we have to consider the set W of classes $\mathbf{G}_+\backslash\mathbf{G}^{\mathbb{C}}/\mathbf{G}_-$ and pick up a representative w for each class $[w]\in W$ (which we shall assume in the following to be discret set). It gives us the following stratification of $\mathbf{G}^{\mathbb{C}}$ [26]:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W}\mathbf{G}_+ w\mathbf{G}_- = \bigcup_{[w]\in W}\mathbf{G_w} \tag{24}$$

There is the second stratification:

$$\mathbf{G}^{\mathbb{C}} = \bigcup_{[w]\in W}\mathbf{G}_- w\mathbf{G}_+ = \bigcup_{[w]\in W}\mathbf{G^w} \tag{25}$$

We shall assume that the representatives w have picked up to satisfy the unitarity condition:

$$\tau(w) = w^{-1} \tag{26}$$

This allows us to generalize (16), (18) as follows

$$g = g_+ w g_-^{-1} = \tilde{g}_- w\tilde{g}_+^{-1}, \tag{27}$$

where $g_\pm$ related with $\tilde{g}_\pm$ by the formula (18). The corresponding generalization of (21) allows us to write down the following generalization of (22)

$$S_{swz} = -\frac{1}{2}\int d^2xd^2\Theta(<\rho_+^+, w\rho_-^- w^{-1}> + <\tilde{\rho}_+^-, w\tilde{\rho}_-^+ w^{-1}>) \tag{28}$$

It is clear that the formula (28) is correct inside the world sheet domain where the superfields take values in the class $\mathbf{G_w}$. On the boundaries of these domains, where the jumps from one class to another is appeared some additional terms should be added, but for our purposes it will suffice to consider the mappings when the whole world-sheet maps into the one of the classes from W.

The formulas (19) ((20)), (27), (28) mean that there is a natural action of the complex group $\mathbf{G}_+$ ($\mathbf{G}_-$) on $\mathbf{G}$, and the set W parametrizes $\mathbf{G}_+$-orbits ($\mathbf{G}_-$-orbits) $\mathbf{G}\cap\mathbf{G_w}$. It is easy to see also that the group $\mathbf{G}^{\mathbb{C}}$ plays the role of Drinfeld's double for the compact group with the complex structure $(\mathbf{G}, J)$. Therefore it would be well to represent a supersymmetric generalization of an off shell PL T-duality formulation using this double. Although the off shell formulation is known in bosonic case [10], [11] its supersymmetric generalization is not straightforward and is still an open problem. Consequently we shall work with the formulas (19, 21, 26, 27, 28) and propose an on shell supersymmetric version of PL T-duality in N=2 SWZNW models on the compact groups considering the simplest example- N=2 $U(2)$-SWZNW model.

3. Poisson-Lie T-duality in N=2 $U(2)$-SWZNW model.

3.1. The complex structure, Manin triple and double Lie group for $U(2)$

Let's fix the standard basis in the Lie algebra $\mathbf{g}$ of the group $U(2)$:

$$\sigma_0 = \begin{pmatrix} \imath & 0 \\ 0 & \imath \end{pmatrix},\ \sigma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & \imath \\ \imath & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} \imath & 0 \\ 0 & -\imath \end{pmatrix} \tag{29}$$

We introduce the complex structure J on $\mathbf{g}$ by the formulas

$$J\sigma_0 = \sigma_3,\ J\sigma_1 = \sigma_2 \tag{30}$$

and define its $\pm\imath$-basics eigenvectors:

$$e^0 = \frac{1}{2}(1 - \imath J)\sigma_0,\ e^1 = \frac{1}{2}(1 - \imath J)\sigma_1 \tag{31}$$

$$e_0 = \frac{1}{2}(1 + \imath J)\sigma_0,\ e_1 = \frac{1}{2}(1 + \imath J)\sigma_0 \tag{32}$$

Let's define the following subspaces in the complexification $\mathbf{g}^{\mathbb{C}}$ of the Lie algebra $\mathbf{g}$

$$\mathbf{g}_- = \mathbb{C}e^0 + \mathbb{C}e^1,\ \mathbf{g}_+ = \mathbb{C}e_0 + \mathbb{C}e_1 \tag{33}$$

Thus $\mathbf{g}^{\mathbb{C}} = \mathbf{g}_- \oplus \mathbf{g}_+$ as a vector space and $\mathbf{g}_\pm$ are maximaly isotropic subalgebras with respect to the inner product $<,>$ on $\mathbf{g}^{\mathbb{C}}$ defined by

$$< x, y >= Tr(xy), \tag{34}$$

where $x, y \in \mathbf{g}^{\mathbb{C}}$. Therefore we have obtained the Manin triple

$$(\mathbf{g}^{\mathbb{C}}, \mathbf{g}_-, \mathbf{g}_+) \tag{35}$$

so that the vectors (31, 32) constitute the orthonormal basis in $\mathbf{g}^{\mathbb{C}}$.

We parametrize the elements from the exponential subgroups $\mathbf{G}_\pm = Exp(\mathbf{g}_\pm)$ by the matrices

$$g_+ = \begin{pmatrix} a_+ & 0 \\ -a_+ b_+ & a_+^{-1} \end{pmatrix} a_+^{-\imath},\ g_- = \begin{pmatrix} a_- & a_-^{-1} b_- \\ 0 & a_-^{-1} \end{pmatrix} a_-^{\imath}, \tag{36}$$

where

$$a_\pm = \exp(\mp z_\pm / 2) \tag{37}$$

(in the following we shall use the same notations: $a_\pm, a_\pm, z_\pm$ for the matrix elements of the superfields $G_\pm$). We denote by $(\mathbf{G}^{\mathbb{C}}, \mathbf{G}_+, \mathbf{G}_-)$ the double Lie group , where

$$\mathbf{G}^{\mathbb{C}} = \mathbf{G}_{\mathbf{1}}^{\mathbb{C}} \cup \mathbf{G}_{\mathbf{s}}^{\mathbb{C}} \sim Gl(2, C),$$
$$\mathbf{G}_{\mathbf{1}}^{\mathbb{C}} = \mathbf{G}_+ \mathbf{G}_-,\ \mathbf{G}_{\mathbf{s}}^{\mathbb{C}} = \mathbf{G}_+ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{G}_-. \tag{38}$$

The solution of the equations (19) are given by

$$z_- = -\bar{z}_+ + \ln(1 + \mid b_+ \mid^2),\ b_- = -\exp(z_+ - \bar{z}_+)\bar{b}_+ \tag{39}$$

and the solution of the equations (16) are given by

$$\begin{aligned} \tilde{z}_+ &= -z_+ + \ln(1 + \mid b_+ \mid^2),\ \tilde{z}_- = \bar{z}_+ \\ \tilde{b}_+ &= -\exp(\bar{z}_+ - z_+) b_+,\ \tilde{b}_- = \bar{b}_+ \end{aligned} \tag{40}$$

Using these formulas we can represent the action (22) in the components

$$S_{swz} = -\frac{1}{2} \int d^2 x d^2 \Theta (E_{ij} \rho_+^i \rho_-^j +\ E_{i\bar{j}} (\rho_+^i \rho_-^{\bar{j}} - \rho_-^i \rho_+^{\bar{j}}) +\ E_{\bar{i}\bar{j}} \rho_+^{\bar{i}} \rho_-^{\bar{j}}), \tag{41}$$

where $i, j = 0, 1$,

$$E_{01} = -E_{10} = -(1+ \mid b_+ \mid^2)^{-1} a_+^{-2} \bar{b}_+, \; E_{\bar{i}\bar{j}} = \bar{E}_{ij}$$
$$E_{0\bar{0}} = 1, \; E_{0\bar{1}} = -(1+ \mid b_+ \mid^2)^{-1} \bar{a}_+^{-2} b_+$$
$$E_{1\bar{0}} = -(1+ \mid b_+ \mid^2)^{-1} a_+^{-2} \bar{b}_+, \; E_{1\bar{1}} = (1+ \mid b_+ \mid^2)^{-1} a_+^{-2} \bar{a}_+^{-2}, \tag{42}$$

other elements of the bilinear form E are zeroes and ρ^i are given by

$$\rho^0 = Dz_+, \; \rho^1 = a_+^2 Db_+ \tag{43}$$

(the fields ρ^i correspond to the components of the left invariant 1-form $g_+^{-1} dg_+$ on the group $\mathbf{G}_+$). For the mappings into the class $\mathbf{G}_{\mathbf{s}}^{\mathbb{C}}$ the action (28) is identicaly zero because

$$< \rho_+^+, w\rho_-^- w^{-1} >= < \tilde{\rho}_+^-, w\tilde{\rho}_-^+ w^{-1} >= 0. \tag{44}$$

3.2. Poisson-Lie symmetry conditions.

In view of the formulas (41-44) we can consider $U(2)$-SWZNW model as a σ-model on the complex Lie group $\mathbf{G}_+$ and find the equations of motion making a variation of the action (41) under the right action of this group on itself.

The right translations on the group $\mathbf{G}_+$ are generated by the vector fields

$$S_0 = \frac{\partial}{\partial z_+}, \; S_1 = \exp(z_+) \frac{\partial}{\partial b_+} \tag{45}$$

and its complex conjugated $\bar{S}_i, i = 0, 1$. Note that the second vector field from (45) coincides with the classical screening current in the Wakimoto representations of $\hat{sl}(2)$ [27]. We would like to stress the general feature of this observation: the screening currents in the Wakimoto representations are given by the right action of the maximal nilpotent subgroup $\mathbf{N}_+$ on the big cell of the corresponding flag manifold $\mathbf{H}/\mathbf{B}_-$, where $\mathbf{B}_-$ is the Borelian subgroup of the group $\mathbf{H}$, $(\mathbf{N}_+ \in \mathbf{B}_+)$ [28], consequently the natural action of the group $\mathbf{G}_+$ in any N=2 SWZNW model on the group $\mathbf{G}$ associated with the Bruhat decomposition is generated by the screening currents.

Making a variation of (41) under the vector field $Z = Z^i S_i + Z^{\bar{i}} \bar{S}_i$ we obtain on the extremals

$$D_+(A_-)_i + D_-(A_+)_i - L_{S_i} E = 0$$
$$D_+(A_-)_{\bar{i}} + D_+(A_+)_{\bar{i}} - L_{\bar{S}_i} E = 0, \tag{46}$$

where $L_{S_i}, L_{\bar{S}_i}$ mean the Lie derivatives along the vector fields $S_i, \bar{S}_i$ and the Noether currents $A_i, A_{\bar{i}}$ are given by

$$(A_-)_i = E_{i\bar{j}} \bar{\rho}_-^j + E_{ij} \rho_-^j,$$
$$(A_+)_i = -E_{i\bar{j}} \bar{\rho}_+^j - E_{ji} \rho_+^j,$$
$$(\bar{A}_\pm)_i = (A_\pm)_{\bar{i}} \tag{47}$$

By the direct calculation we can check that the following PL symmetry conditions are satisfied on the extremals

$$L_{S_i} E = f_i^{nm} (A_+)_n (A_-)_m$$
$$L_{\bar{S}_i} E = \bar{f}_i^{nm} (A_+)_{\bar{n}} (A_-)_{\bar{m}}, \tag{48}$$

where f_i^{nm} are the structure constants of the Lie algebra $\mathbf{g}_-$. By demanding the closure of (48): $[L_{S_i}, L_{S_j}] = f_{ij}^k L_{S_k}$, we obtain the following consistency condition

$$f_{ij}^n f_n^{km} = f_j^{nm} f_{in}^k - f_j^{nk} f_{in}^m \ - f_i^{nm} f_{jn}^k + f_i^{nk} f_{jn}^m \tag{49}$$

which is satisfyed due to the Jacoby identity in the Lie algebra $\mathbf{g}^{\mathbb{C}}$.

As it is easy to see from (46) the eq. (48) are equivalent to the zero curvature equations for the F_{+-}-component of the super stress tensor F_{MN}

$$\begin{aligned}(F_{+-})_i &\equiv D_+(A_-)_i + D_-(A_+)_i - f_i^{nm}(A_+)_n(A_-)_m = 0\\ (F_{+-})_{\bar{i}} &\equiv D_+(A_-)_{\bar{i}} + D_-(A_+)_{\bar{i}} - \bar{f}_i^{nm}(A_+)_{\bar{n}}(A_-)_{\bar{m}} = 0\end{aligned} \tag{50}$$

Using the standard arguments of the super Lax construction [29] one can show that from (50) it follows that the connection is flat

$$F_{MN} = 0,\ M, N = (+, -, +, -). \tag{51}$$

The equations (50) are the supersymmetric generalization of Poisson-Lie symmetry conditions from the work [8]. Indeed, the Noether currents $A_i, A_{\bar{i}}$ are generators of $\mathbf{g}_+$- action, while the structure constants in (50) correspond to Lie algebra $\mathbf{g}_-$ which is Drinfeld's dual to $\mathbf{g}_+$ [31].

3.3. Poisson-Lie T-dual σ-model.

The PL-dual to $U(2)$-SWZNW σ-model should obey the conditions as (48) but with the roles of the Lie algebras $\mathbf{g}_\pm$ interchanged [8].

To construct this σ-model we associate (due to (51)) to each solution $G_+(x_+, x_-, \Theta_+, \Theta_-)$, the map $V_-(x_+, x_-, \Theta_+, \Theta_-)$ from the super world-sheet into the group $\mathbf{G}_-$ such that

$$(A_\pm)_i e^i = D_\pm V_- V_-^{-1} \tag{52}$$

From the other hand, using (39, 42) one can obtain

$$\begin{aligned}(A_-)_i e^i &= -\rho_-^-\\ (A_+)_i e^i &\neq \pm\rho_+^-\end{aligned} \tag{53}$$

Therefore one can represent V_- as the product

$$V_- = G_-^{-1} H_-^{-1} \tag{54}$$

, where G_- is determined from (21) and H_- satisfy the equation

$$D_- H_- = 0 \tag{55}$$

Now we build the following surface in the Drinfeld's double $\mathbf{G}^{\mathbb{C}}$:

$$\begin{aligned}F(x_+, x_-, \Theta_+, \Theta_-) &= G_+(x_+, x_-, \Theta_+, \Theta_-) V_-(x_+, x_-, \Theta_+, \Theta_-)\\ &= G(x_+, x_-, \Theta_+, \Theta_-) H_-(x_+, x_-, \Theta_+, \Theta_-),\end{aligned} \tag{56}$$

where $G_+(x_+, x_-, \Theta_+, \Theta_-), G(x_+, x_-, \Theta_+, \Theta_-)$ are the classical solutions of $U(2)$-SWZNW model, $V_-(x_+, x_-, \Theta_+, \Theta_-)$ satisfy (52) and we have used (54). The solution of the dual σ-model is given by "dual" parametrization of the surface (56) [8]

$$\begin{aligned} F(x_+, x_-, \Theta_+, \Theta_-) &= \breve{G}_-(x_+, x_-, \Theta_+, \Theta_-)\breve{V}_+(x_+, x_-, \Theta_+, \Theta_-) \\ &= \breve{G}(x_+, x_-, \Theta_+, \Theta_-)\breve{H}_+(x_+, x_-, \Theta_+, \Theta_-), \end{aligned} \tag{57}$$

where $\breve{G}(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}$ and $\breve{H}_+(x_+, x_-, \Theta_+, \Theta_-) \in \mathbf{G}_+$. In the dual σ-model Drinfeld's dual group to the group $\mathbf{G}_+$ should acts, i.e. it should be a σ-model on the orbits of the group $\mathbf{G}_-$. Because the hermitian conjugation τ conjugates subgroups $\mathbf{G}_\pm$ and the action (28) is real we conclude that the dual σ-model should coincide with the initial one, thus N=2 $U(2)$-SWZNW model is PL self-dual.

5. Conclusions.

We have shown that the classical N=2 $U(2)$-SWZNW model possess very natural PL symmetry with respect to the isotropic subgroups $\mathbf{G}_\pm$ forming the Drinfeld's double $Gl(2, C)$. The infinitesimal action of the group $\mathbf{G}_+$ ($\mathbf{G}_-$) are given by the classical screening currents in the Wakimoto representations of the $\hat{u}(2)$-current algebra. PL T-dual σ-model to N=2 $U(2)$-SWZNW model is N=2 $U(2)$-SWZNW itself but the PL T-duality transformation acts nontrivialy on the classical solutions.

We have shown also that the role of Drinfeld's double in N=2 SWZNW model on an arbitriary compact group $\mathbf{G}$ plays its complexification $\mathbf{G}^{\mathbb{C}}$ endowed with the hermitian conjugation which conjugates isotropic subgroups. There is the natural action of isotropic subgroups $\mathbf{G}_\pm \in \mathbf{G}^{\mathbb{C}}$ on $\mathbf{G}$ generated by the corresponding screening currents. Having established PL symmetry in the simplest case of N=2 $U(2)$-SWZNW model it would appear reasonable that *N=2 SWZNW models on the compact groups of higher dimensions admit PL symmetries with respect to the actions of isotropic subgroups from the corresponding Drinfeld's doubles such that these models are PL T-selfdual.* We will publish the proof of this proposal in the nearst future [30]. We expect the PL T-duality exists also in N=2 superconformal Kazama-Suzuki models [32] since these models can be represented as the cosets (N=2 $\mathbf{G}$-SWZNW model)/(N=2 $\mathbf{H}$-SWZNW model), where $\mathbf{H}$ is a subgroup from $\mathbf{G}$ [33].

It is an intresting question what is the quantum picture of the PL T-duality in N=2 SWZNW models. Because the Poisson-Lie groups is nothing but a classical limit of the quantum groups [9] there appears an intriguing possibility of a relevance of quantum groups in the T-duality and other superstring applications for example in D-branes [12], [34].

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